Infinite geometric series practice

1. Let $\{u_n\}$ be an arithmetic sequence with first term equal to a and the common difference of d. Let another sequence be defined by $v_n = 3^{u_n}$

   (a) Find v_n in terms of a, d, and n

   (b) Find S_n in terms of a, d, and n where S_n be the sum of the first n terms of the sequence $\{v_n\}$

   Let $S = \sum_{k=1}^{\infty} v_k$

   (c) Find d satisfies S is converged.

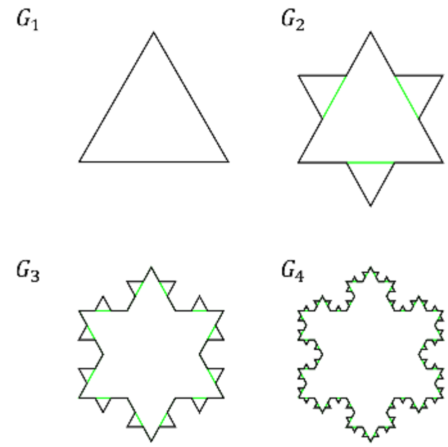
   (d) Given that $S = 3^{a+1}$, find d.

   (e) Let $\{w_n\}$ be a geometric sequence with first term equal to p and the common ratio of q where $p, q > 0$. Let another sequence $\{z_n\}$ be defined by $z_n = \ln w_n$.

   Find $\sum_{i=1}^{n} z_i$ giving your answer in the form $\ln k$ in terms of p, q, amd n.

2. (Von Koch's snowflake) Consider a equilateral triangle with length 1 (denote G_1). We construct a new small equilateral for each side of G_1 to construct G_2. The length of the small triangle is one-third of the lenght of the side. Use the same pattern, we can construct $G_3, G_4, \cdots$.

(a) Write down the area of G_1, G_2, and G_3
(Do not simplify)



(b) Observe the pattern, Find the area of G_n

(c) Hence, find $\lim_{n\to\infty} G_n$

3. (Sierpinski triangle) Observe the pattern. Let G_n represent the area of the black region. Suppose the first graph G_1 is an equilateral triangle with length 1. Find $\lim_{n\to\infty} G_n$